Scholarly Research Journal for Interdisciplinary Studies,
Online ISSN 2278-8808, SJIF 2016 = 6.177, www.srjis.com
UGC Approved Sr. No.49366, MAR–APR, 2018, VOL- 5/44

DESIGN OF DYNAMIC VIBRATION ABSORBER FOR MINIMIZING THE AMPLITUDE MAGNIFICATION FACTOR

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Abstract

*Now-a-days in daily life mechanical equipment are used for our sustainable needs. In mechanical equipment vibration absorbers play important role in the study of suspension system. During the working condition of vibration absorber, it may fail due to sudden loads, material properties. The main purpose of this research is to analyze the dynamic vibration absorber with a modification in the primary system and to define whether the vibration absorber is better compared to the previous model. By using MATLAB, the vibration absorber system is developed and defined the parameters for different conditions and got the output in frequency (Hz) and amplitudes (**m**) and compared the results of the previous model with the present model by building the system using the governing equations in MATLAB. Similarly dynamic vibration absorber is designed with some parameters through drafting software by using solid works. In Ansys, a vibration absorber is designed and imported from solid works by applying the load condition, thermal condition and number of elements. Finally, at different impact loads the frequencies and the respective amplitudes are obtained. Having known the frequencies and amplitude behavior, the performance of dynamic vibration absorber can be estimated.*

INTRODUCTION

When an elastic body such as spring, beam and shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a vibratory motion, due to the elastic or strain energy present in the body. When the body reaches the equilibrium position, the whole of the elastic or stain energy is converted into kinetic energy due to which the body continues to move in the opposite direction. The entire KE is again converted into strain energy due to which the body again returns to the equilibrium position. Hence the vibratory motion is repeated indefinitely. Vibratory systems comprise means for storing potential energy (spring), means for storing kinetic energy (mass or inertia), and means by which the energy is gradually lost (damper). The vibration of a system involves the alternating transfer of energy between its potential and kinetic forms. In a damped system, some energy is dissipated at each cycle of vibration and must be replaced from an external source if a steady vibration is to be maintained. Although a single physical structure may store both kinetic and potential energy, and may dissipate energy, this chapter considers only lumped parameter systems composed of ideal springs, masses, and dampers wherein each element has only a single function. In translational motion, displacements are defined as linear distances; in rotational motion, displacements are defined as angular motions.

Osamu Nishihara [1] has carried out the minimization of the maximum amplitude magnification factor of a three-element DVA. Then, the effects of this formulation were precisely evaluated based on the equivalent mass ratio, which is defined as the mass ratio of the Voigt DVA that realizes the same amplitude magnification factor. It seems that for a larger mass ratio, for example, for a ratio of more than 0.1, the addition of a spring to the popular Voigt DVA in series with a viscous damper is considerably effective if the parameter values can be adjusted as shown herein. This concept would be especially effective in cases for which a large-scale DVA is designed for a heavy primary system. The approach proposed here realizes rapid convergence to the exact solution and readily obtains highly precise solutions. Toshihiko Asami, Yoshito Mizukawa, Tomohiko Ise [2] performed analysis on the basis of three design criteria, double-mass vibration absorbers arranged in series or in parallel were optimized to minimize the mobility transfer function of the primary vibratory system. A comparison of the present results with a previously published solution minimizing the compliance transfer function yielded the conclusions they are, the algebraic solution for the optimization of a double-mass DVA was successfully obtained for the first time, In the minimization of the compliance transfer function, the optimal value of the damping ratio of absorber in the series-type DVA was always zero. J. C. Snowdon [3] has investigated effectiveness of a three-element dynamic absorber and of dual dynamic absorbers in reducing the transmissibility across a simple spring-mass system at resonance has been investigated and shown to be considerably greater than that of the conventional dynamic absorber. From the complete information provided, much in graphical form, it is possible to design and to estimate the performance of both the three-element and the dual dynamic absorbers for a wide range of absorber masses.

Utilizing design parameters that have been determined and specified here for the first time, it is shown that the three element absorber can be more effective than a conventional absorber of twice its mass, and that by use of dual absorbers. The three-element absorber requires no increase, and the dual absorbers require only a modest increase in mass beyond that of the conventional dynamic absorber.

W.O. Wong, R. P. Fan and F. Cheng[4] has proposed VDVA for suppressing vibrations of heavy mechanical or civil structures. The stiffness and damping of the proposed VDVA can be decoupled such that both of these two properties of the absorber can be tuned independently to their optimal values by following a specified procedure. A modified fixed-points theory is therefore proposed to solve this problem. Optimization of the proposed VDVA have been derived analytically for the minimization of resonant vibration of a system excited by harmonic forces or due to ground motions. Simple analytical expressions of the optimal additional stiffness and geometric factor of the proposed VDVA are derived using the modified fixed-points theory. The proposed VDVA with optimized design is tested numerically using two real commercial viscoelastic damping materials. It is found that the proposed viscoelastic absorber can provide much stronger vibration reduction effect than the conventional VDVA without the elastic spring. The proposed optimal design methodology of dynamic vibration absorber may help engineers to suppress infrasonic vibrations of heavy structures and the proposed VDVA may be considered as an alternative design of the traditional DVA as well. In this work the authors Toshihiko Asami, Osamu Nishihara[5] state that the dynamic vibration absorber (DVA) is a passive vibration control device which is attached to a vibrating body (called a primary system) subjected to exciting force or motion. Kefu Liu, Jie Liu[6] have failed to obtain the results using Den Harto's method and Kelly's method. After comparing Brock's approach with the previous methods, they have realized that Brock employed a perturbation method instead of differentiating a high-order equation. They have applied Brock's approach to a different type of damped vibration absorber. They have found the optimum parameters. They have verified all the results numerically and presented a comparison of the two models. Toshihiko Asami, Osamu Nishihara [7] have shown that the algebraic exact solution exists for the minimization of the resonance amplitude magnification factor by the dynamic vibration absorber attached to the undamped primary system. The exact algebraic expressions for the resonance and anti-resonance frequencies have been obtained. The local optimality of the solution then became apparent. This approach was extended to the damped primary case, but complementary numerical solution. The fixed-points method was shown to be highly accurate, especially for small mass ratios of less than or around unity. This method is applicable to the linearized model of the damper. Their algebraic solution of optimum parameters has been obtained under the assumption of undamped primary system which is modeled by a three-element type system. New expressions for the optimum parameters have been derived. Mohammad Salavati [8] have made a study, a model-based structural damping identification method is proposed. The idea behind this proposed method is a theoretical comparison of the damped and un-damped system models. In the second part, estimating of the input excitation force is considered. This method is based on the fundamental concept of the FRF. The effectiveness of them is investigated by numerical and analytical approaches. The importance of these methods is their ability to identifying target parameters just by using measured responses and also for each frequency content of a response it can be possible to track each SDOFs behavior. In addition, capability of proposed damping identification method in different modeling of the damping or following the damping variation of the structure in the case of unknown damping model. The identified responses and input excitation force by using these methods are compared with reference ones that are used in the simulation process. High convergence results illustrate the satisfactory of the proposed approximation. Lei Zuo, Samir A. Nayfeh [9] have proposed the use of a MDOF TMD for one mode of primary system and show that, for a given mass, 2DOF TMD performs better than a traditional SDOFTMD or two separate TMDs with optimal mass distribution. We cast the parameter optimization of MDOF TMD systems as a decentralized control problem, where the block-diagonal controller gain is directly composed of the stiffness and damping parameters of the connections between the absorber and primary system. Based on this formulation, we adapt decentralized H2 and H-optimization techniques to optimize the system response under random and harmonic excitation, respectively. First, we employ gradient-based decentralized H2 optimization to minimize the RMS response under random excitation and provide a comprehensive study of the performance of a 2DOF TMD attached to a SDOF primary system. Design charts for passive TMD implementation in which all of the springs and dampers are required to be positive are provided. We then discuss the case where the dampers are allowed to be negative, and find that the performance is considerably improved. This suggests that an effective reaction-mass actuator can be constructed with a 2DOFreaction mass. We propose an algorithm for decentralized H_ optimization to minimize the peak of the frequency response under harmonic excitation. The maximal response is obtained efficiently using-iteration and finite differences are used to approximate its gradient with respect to the design parameters. We then optimize the2DOF absorber and find that its frequency-domain performance is again better than that of the SDOF absorber or two SDOF absorbers.

Lei Zuo, Samir A. Nayfeh[10] In this paper the author states that Multi-degree-of-freedom (MDOF) tuned-mass-dampers (TMD) can be tuned to damp more than one mode of a primary system efficiently. In this paper, the problem of designing a MDOF TMD attached to a MDOF primary system is formulated as a decentralized static-output feedback problem. Then an e-sub gradient algorithm is presented that maximizes the minimum Damping over a prescribed frequency range in order to obtain the optimal parameters of the MDOF TMD. In this approach, we can impose constraints on the ranges of the parameters and design for marginally stable and hysteretically damped systems directly. Lei Zuo[11] In this paper the author has studied a new type of tuned-mass damper: series multiple TMDs. The author has demonstrated that significantly increased effectiveness and robustness can be obtained when absorbers are attached to the primary system in series. He optimize the parameters of stiffness and damping of series multiple TMDs using decentralized *H_* and *H*2 control methods for harmonic and random vibration. Then we thoroughly investigate the characteristics of series TMDs composed of two absorbers. The mass distribution among the two absorbers in series is very important to minimize the peak magnitude or the rms value of the primary system. At the optimum, the first absorber is generally larger than the second absorber, the first tuning ratio is larger than the second, and the first dashpot is zero. If the first absorber mass is relatively small, the performance is close to _and just slightly better than_ the three- or four-element TMD. When the first absorber is much larger than the second absorber, the optimal damping of the first dashpot is not zero. It is interesting to find that the first tuning ratio of the optimal series two TMDs with optimal mass distributions is always 1 under random vibration. Optimal performances and parameters are obtained and presented in a chart form ready to use in the design of series two TMDs for both harmonic and random vibration. Series TMDs can be much more effective and more robust than the parallel TMDs, the multi-DOF TMD, the three- or four element TMD, and the classic TMD. Series two TMDs of a total mass ratio of 5% can achieve the minimal peak magnitude of the classic TMD of a mass ratio of 8.3%, or of the the classic TMD of a mass ratio of 5% plus 2.6% primary damping. For random vibration, series two TMDs of a total mass ratio of 5% can achieve the effectiveness of the classic TMD of a mass ratio of 6.6%. Two TMDs in series can be more effective than ten TMDs attached to the primary system in parallel. Series two TMDs can be better than the optimal symmetric 2DOF TMD for both random and harmonic vibration. The series two TMDs at optimum is almost insensitive to the mass change of the primary system and is less sensitive to the parameter change of the primary system than the other types of TMDs. The series two TMDs are also robust to parameter change in the absorbers. Kouichi IWANAMI, Kazuto SETO[12] Inthispaper, theconditional equations for the optimum adjustment which werederivedbyanalyzing the optimumadjusting conditions for the dual dynamic chartofcomputerprogramtocalculatethe optimumadjustment values from the equations,thepracticalapproximateequations withonlyone parameteru', themethod of the optimum adjustment and the charts I or the optimum adjustment are pre sented. From the analysis described above, the fo1lowing are concluded Thee dual dynamic damper can reduce the decrease of the vibration control effect,which is a drawback toan ordinary dynamic damper, caused by the change of the natural frequency in themainvibration system. Asystemwithtwodampersconnected, thatis, a dual dynamic damper, enables anadjustment allowing for the changeof thenatural frequency in the main vibration system. The influence of the change of the damping coefficient O is less in the dual dynamic damperthan in the singledamper.

In this research work in addition to the work published by Osamu Nishihara [1] a damper is attached in parallel to the spring in the primary system of three element dynamic vibration absorber. The results obtained are better for the modified system when compared with the results obtained for the system used by Osamu Nishihara.

2.1 Theoretical Vibration Analysis Procedure:

A vibratory system is a dynamic one for which the variables such as the excitations and responses are time dependent. The response of a vibrating system generally depends on the initial conditions as well as the external excitations. Most practical vibrating systems are very complex, and it is impossible to consider all the details for a mathematical analysis. Only the most important features are considered in the analysis to predict the behavior of the system under specified input conditions. Often the overall behavior of the system can be determined by considering even a simple model of the complex physical system. Thus the analysis of a vibrating system usually involves mathematical modeling, derivation of the governing equations, solution of the equations, and interpretation of the results.

2.1.1 Step 1: Mathematical Modeling.

The purpose of mathematical modeling is to represent all the important features of the system for the purpose of deriving the mathematical (or analytical) equations governing the system behavior. The mathematical model should include enough details to allow describing the system in terms of equations without making it too complex. The mathematical model may be linear or nonlinear, depending on the behavior of the system s components. Linear models permit quick solutions and are simple to handle; however, nonlinear models sometimes reveal certain characteristics of the system that cannot be predicted using linear models. Thus a great deal of engineering judgment is needed to come up with a suitable mathematical model of a vibrating system. Sometimes the mathematical model is gradually improved to obtain more accurate results. In this approach, first a very crude or elementary model is used to get a quick insight into the overall behavior of the system. Subsequently, the model is refined by including more components and/or details so that the behavior of the system can be observed more closely.

2.1.2 Step 2: Derivation of Governing Equations.

Once the mathematical model is available, we use the principles of dynamics and derive the equations that describe the vibration of the system. The equations of motion can be derived conveniently by drawing the free-body diagrams of all the masses involved. The free-body diagram of a mass can be obtained by isolating the mass and indicating all externally applied forces, the reactive forces, and the inertia forces. The equations of motion of a vibrating system are usually in the form of a set of ordinary differential equations for a discrete system and partial differential equations for a continuous system. The equations may be linear or nonlinear, depending on the behavior of the components of the system. Several approaches are commonly used to derive the governing equations. Among them are Newton s second law of motion, D-Alembert s principle, and the principle of conservation of energy.

2.1.3 Step 3: Solution of the Governing Equations.

The equations of motion must be solved to find the response of the vibrating system. Depending on the nature of the problem, we can use one of the following techniques for finding the solution: standard methods of solving differential equations, Laplace transform methods, matrix methods and numerical methods. If the governing equations are nonlinear, they can seldom be solved in closed form. Furthermore, the solution of partial differential equations is far more involved than that of ordinary differential equations. Numerical methods involving computers can be used to solve the equations. However, it will be difficult to draw general conclusions about the behavior of the system using computer results.

2.1.4 Step 4: Interpretation of the Results.

The solution of the governing equations gives the displacements, velocities, and accelerations of the various masses of the system. These results must be interpreted with a clear view of the purpose of the analysis and the possible design implications of the results.

Example: - To study the dynamic absorber system, a real system considered as main system is modelled as an equivalent single degree of freedom system and it is excited by a harmonic excitation force F= F0 sinωt. The steady state response of the system is given by x= X sin (ωt+φ). Steady state amplitude of vibration of the proposed single degree of freedom system will be maximum at the resonance. To neutralize the effect at resonance, the main system couples with an absorber system. This coupling will affect (suppress) the amplitude of vibration of the main system. By the addition of absorber system, single degree of freedom analysis cannot hold. Hence whole system should be considered two degree of freedom system.

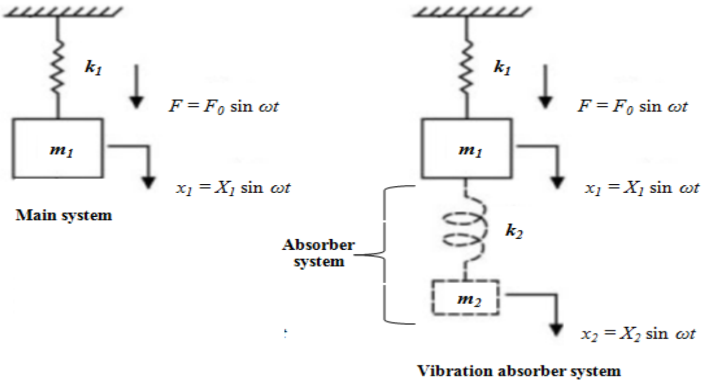

Fig.1 Free body diagram of 2DOF system.

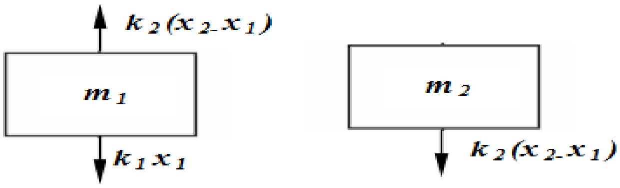

Fig. 2 Free body diagram of vibration absorber system.

The free-body diagrams of the masses and are shown in Fig. 2. By application of Newton's second law of motion to each of the masses gives the equations of motion as:

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = F_0 \sin(\omega t) \quad \text{.... (1)}$$

$$m_2 \ddot{x}_2 + k_2(x_2 - x_1) = 0 \quad \text{.......(2)}$$

Steady state response of two degree of freedom is assumed as,

$$x_1 = X_1 \sin(\omega t + \Phi) \text{ and } x_2 = X_2 \sin(\omega t + \Phi) \quad \text{....(3)}$$

By substituting x1 and x2 in equation 1 and 2 we get,

$$(k_1 + k_2 - m_1 \omega^2)X_1 - k_2 X_2 = F_0 \; -k_2 X_1 + (-m_2 \omega^2 + k_2)X_2 = 0 \quad \text{....(4)}$$

By solving the equation 3 and 4, amplitude of vibration of main system and absorber system given by,

$$X_1 = \frac{(k_2 - m_2 \omega^2)}{[m_1 m_2 \omega^4 - \{m_1 k_2 + m_2(k_1 + k_2)\}\omega^2 + k_1 k_2]} \quad \text{.......(5)}$$

And

$$X_2 = \frac{k_2 F_0}{[m_1 m_2 \omega^4 - \{m_1 k_2 + m_2(k_1 + k_2)\}\omega^2 + k_1 k_2]} \quad \text{......(6)}$$

From equation 5 it can be observed that the amplitude of vibration of main system X1 can be zero if numerator becomes zero.

$$k_2 - m_2 \omega^2 = 0 \quad \text{.....or}$$

$$\omega^2 = \frac{k_2}{m_2} = \omega_{n2}^2 \quad \text{.....(7)}$$

From equation 7, it can be concluded that when the excitation frequency is equal to the natural frequency of the absorber, then main system amplitude becomes zero even though it is excited by harmonic force. Dimensionless form of equation 5 and 6 can be written as

$$\frac{X_1}{X_{st}} = \frac{(1 - \frac{\omega^2}{\omega_2^2})}{\frac{\omega^4}{\omega_1^2 \omega_2^2} - \{(1+\mu)\frac{\omega^2}{\omega_1^2} + \frac{\omega^2}{\omega_2^2}\} + 1} \quad \text{....(8)}$$

$$\frac{X_1}{X_{st}} = \frac{(1 - \frac{\omega^2}{\omega_2^2})}{\frac{\omega^4}{\omega_1^2 \omega_2^2} - \{(1+\mu)\frac{\omega^2}{\omega_1^2} + \frac{\omega^2}{\omega_2^2}\} + 1} \quad \text{......(9)}$$

Equations 8 and 9 give the amplitude response of main system and absorber system as a function of exciting frequency. Where,

$X_{st} = \frac{F_0}{k}$ = Zero frequency deflection of the main system.

ω1 =Natural frequency of the main system.

ω2 =Natural frequency of the absorber system.

ω =Frequency of external excitation.

μ =ratio of absorber mass to the main mass.

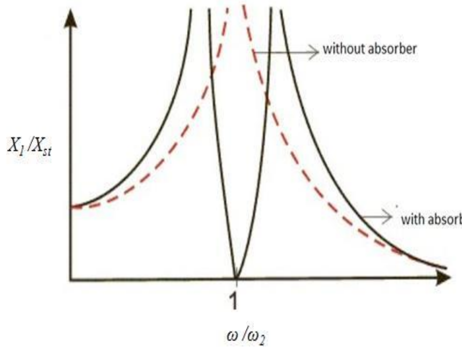

Fig.3 shows the variation of X1/Xst for different frequency ratios.

3. Results

The results obtained by using the MATLAB for the developed model shows that there is better control on the vibrations. The time period to reduce the vibrations or simply in the amplitude reduction is less when compared to the time period to reduce amplitude in the old model. That implies the vibrations are controlled with in the short span. This evaluation is made based on the results from MATLAB and the same are mentioned below.

For the old model or model-1 DVA the plots obtained are

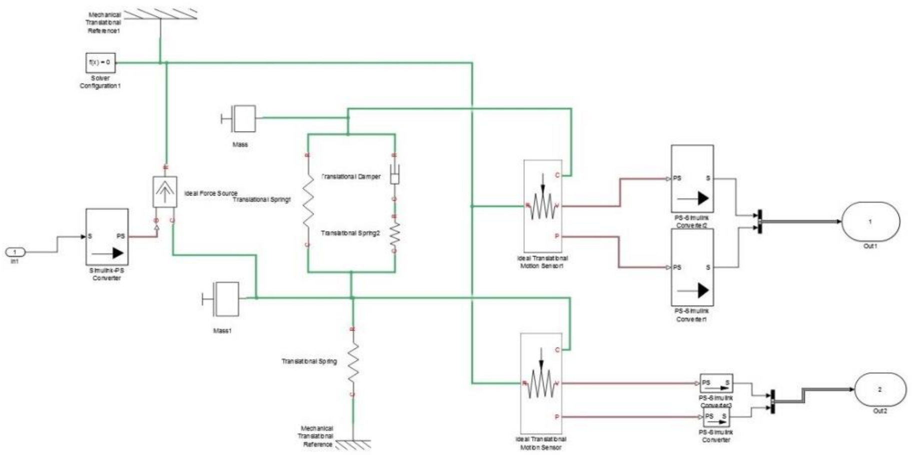

Fig. 4 MATLAB system showing old model DVA

Fig. 4 shows the circuit diagram of Model-1 Dynamic Vibration Absorber which is designed in MATLAB.

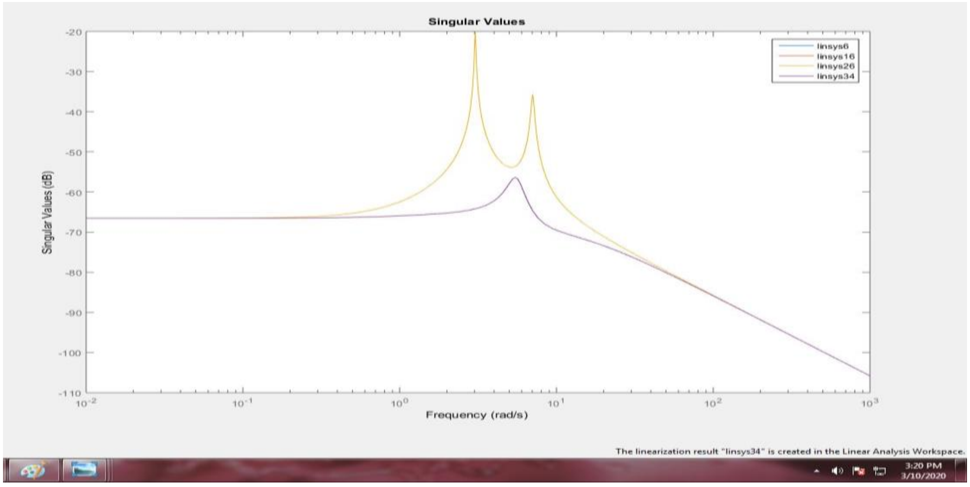

Fig. 5 Frequency curve for model-1

Fig. 5 shows the frequency response curves of Model-1 Dynamic Vibration Absorber which is designed in MATLAB.

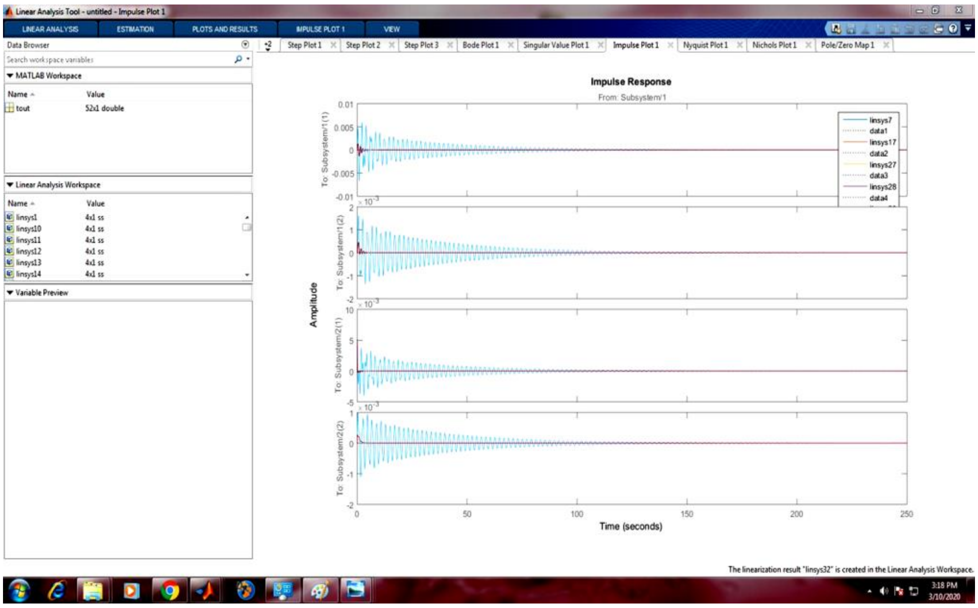

Fig. 6. Step response curves

Fig.6. shows the Impulse response curves of Model-1 Dynamic Vibration Absorber which is designed in MATLAB.

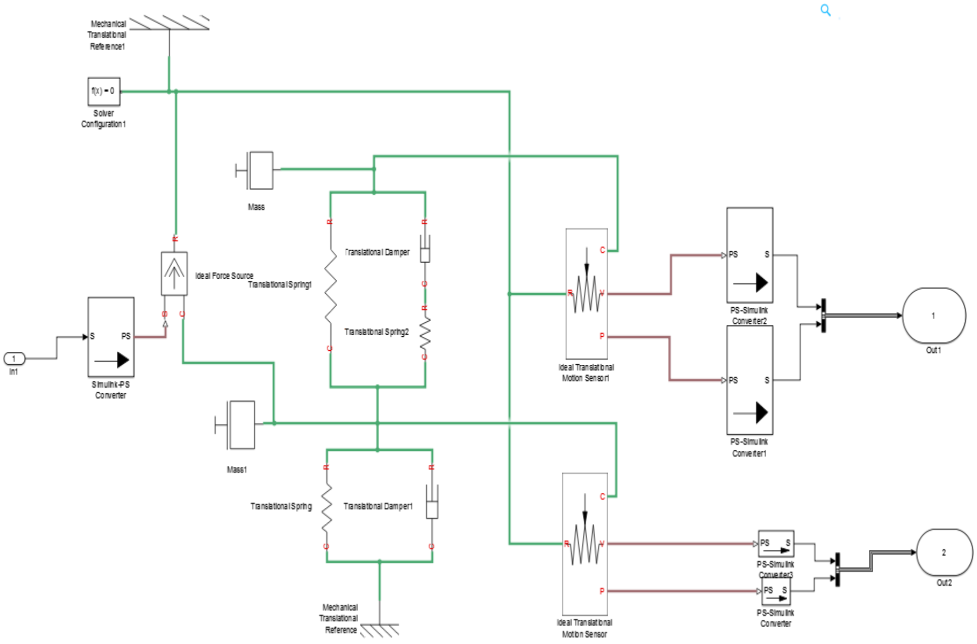

Fig.7. MATLAB system for model-2.

Fig. 7. shows the circuit diagram of Model-2 Dynamic Vibration Absorber which is designed in MATLAB.

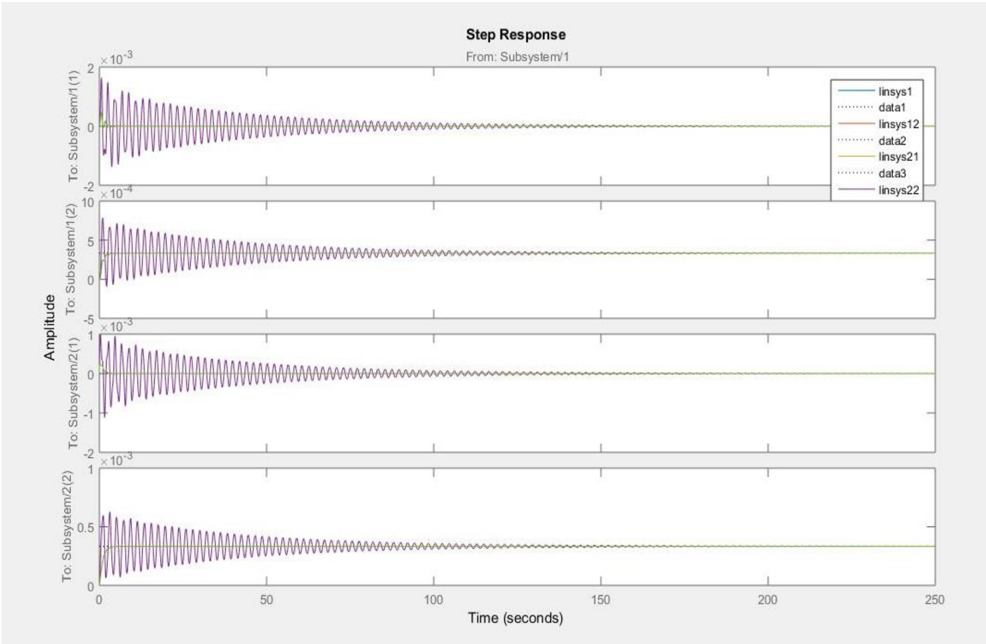

Fig. 8. Step response for model-2.

Fig. 8. shows the step response of Model-2 Dynamic Vibration Absorber which is designed in MATLAB.

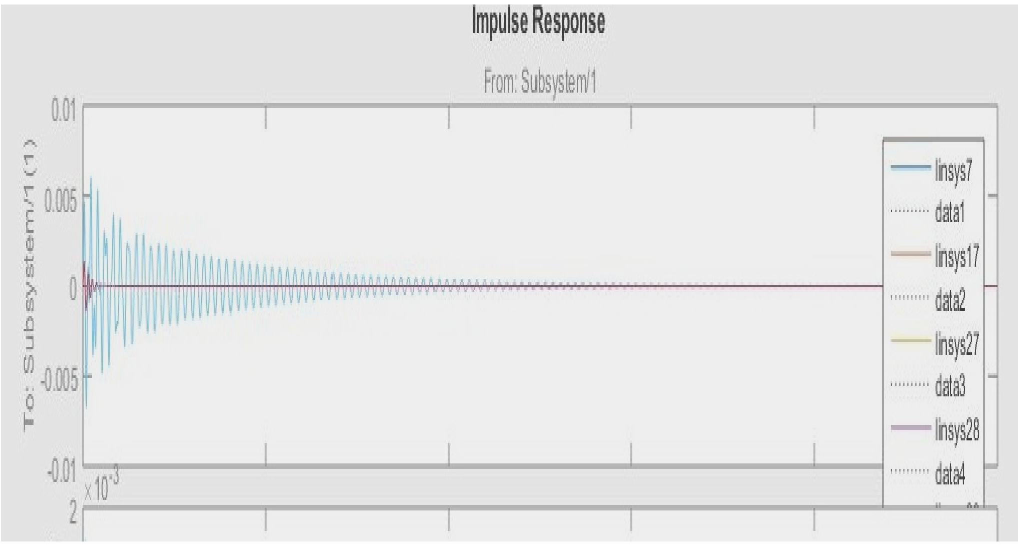

Fig. 9. Impulse response curves

Fig. 9. shows the Impulse response curves of Model-2 Dynamic Vibration Absorber which is designed in MATLAB.

The input values are given in Table 1.

Table.1 input values for model -1&2

Type	m_1	m_2	k_1	k_2	k_a	c_1	c_2
model-1	195kgs	95kgs	3000	1000	500	500	_
model-2	195kgs	95kgs	1500	1000	500	1500	500

Table.2 Step response for model-1&2

Type of system	Step response for model-1		Step response for model-2	
	Amplitude	Time(sec)	Amplitude	Time(sec)
Primary	0.005	50	0.003	40
Secondary	0.0016	5.2	0.001	3.4

On comparing the plots of both model-1 and model-2, it is clearly observed that the for the same amount of force the amplitude in the model-2 is being reduced rapidly with respect to the time when compared to the model-1. The simulation for both the models is also carried out in the Wolfram demonstrations project and the results are further compared to check the effectiveness of the models.

4. DISCUSSION

Using MATLAB, we plot the variations of vibration amplitudes of the main and auxiliary masses of a vibration absorber models as functions of the frequency ratio(g). From the above curve, we see that the amplitude of the absorber mass is always much greater than that of the main mass. Thus the design should be able to accommodate the large amplitudes of the absorber mass. For the model as shown in Figures, as the mass ratio increases amplitude of vibration decreases. As mass ratio increases the optimum damping also increases. One observation we can made from the Frequency response function ‘s curve is the response curve becomes flatter as the mass ratio increases. If we draw the comparison curve of all 2 model in one glance by using MATLAB plot and also using optimal parameters of all the model, we came to know that for model 2 to be optimum, a larger damping is required as compared to model 1. Overall, model 2 gives better vibration suppression and also required damping.

5.CONCLUSIONS

Based on the analysis, the following points have been concluded:

The responses of the main mass and absorber masses have been represented graphically as functions of the frequency ratio by using MATLAB. In order to judge the effectiveness of the parallel vibration absorber, the responses of the conventional absorber are compared with those of the corresponding parallel vibration absorber.After completing the simulation experiment, one should be able to model a given real system to an equivalent simplified single degree of freedom system and reducing the vibration of the main system adding an absorber system with suitable assumptions.

On comparing the plots of both model-1 and model-2, it is concluded that for the same amount of force the amplitude in the model-2 is being reduced rapidly with respect to the time when compared to the model-1.

6.Scope for Prospective Studies

According to the literature, the topic of this study is one of the most important research studies that have been considered in the protective maintenance of Suspension systems. This research topic has a crucial impact on the factors that govern the sustainability of vibration absorption system. The approaches that have been presented in this research study can be considered as a beginning of an idea that will provide further insight into the dynamic Vibration Absorbers. Both the structure and Damper model considered in this study are linear one; this provides a further scope to study this problem using a nonlinear model. The model considered here is two-dimensional, which can be further studied to include 3- dimensional structure model. Further scope, also includes studying the possibility of constructing Active dynamic vibration absorber. Despite of its many advantages for DVA it can be effectively used only in the case of constant speed machines. But in many practical applications the excitation frequency may not be constant: In such cases, the concept of DVA may not be applicable because the absorber is designed with a frequency matches the excitation frequency. For a wide range of excitation frequencies either the mass or stiffness of the spring has made to be varied according to the variation in its excitation frequency. To improve the effectiveness of the conventional absorber by suitable modification, or To invent entirely different and better devices in the hope of replacing the conventional absorber. However, the only modification considered so far in the former group is the addition of damping to the absorber mass. The purpose of this is to examine a further modification of the conventional absorber. Such a modification consists of adding, in parallel, a subsidiary undamped absorber mass in addition to the damped absorber mass.

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